

DO NOW

Check homework.

162. False

$$\frac{1}{4}(x^2 + 1)^2 + C$$

166. False

$$\frac{1}{6}\sin^3 2x + C$$

5.5 Integration by Substitution - Day 4

Definite Integrals

If the function $u = g(x)$ has a continuous derivative on the closed interval $[a, b]$ and f is continuous on the range of g , then:

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

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Evaluate:

$$1. \int_0^1 x(x^2 + 1)^3 dx$$

$$\frac{1}{2} \int_1^2 u^3 du$$

$$\left[\frac{1}{2} \cdot \frac{1}{4} u^4 \right]_1^2$$

$$\frac{1}{8}(2)^4 - \frac{1}{8}(1)^4$$

$$2 - \frac{1}{8}$$

$$\boxed{\frac{15}{8}}$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\text{if } x=0 \rightarrow u=1$$

$$\text{if } x=1 \rightarrow u=2$$

$$2. \int_0^2 x \sqrt[3]{4+x^2} dx$$

$$\frac{1}{2} \int_4^8 u^{1/3} du$$

$$\left[\frac{1}{2} \cdot \frac{3}{4} u^{4/3} \right]_4^8$$

$$\frac{3}{8}(8)^{4/3} - \frac{3}{8}(4)^{4/3}$$

$$6 - 2.3811$$

$$\approx 3.619$$

$$u = 4 + x^2$$

$$du = 2x dx$$

$$x=0 \rightarrow u=4$$

$$x=2 \rightarrow u=8$$

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$$3. \int_1^5 x \sqrt{2x-1} dx$$

$$\frac{1}{2} \int_1^9 \frac{u+1}{2} u^{1/2} du$$

$$\frac{1}{4} \int_1^9 (u+1) u^{1/2} du$$

$$\frac{1}{4} \int_1^9 (u^{3/2} + u^{1/2}) du$$

$$\left[\frac{1}{4} \cdot \frac{2}{5} u^{5/2} + \frac{1}{4} \cdot \frac{2}{3} u^{3/2} \right]_1^9$$

$$\left[\frac{1}{10} u^{5/2} + \frac{1}{6} u^{3/2} \right]_1^9$$

$$\frac{1}{10}(9)^{5/2} + \frac{1}{6}(9)^{3/2} - \frac{1}{10}(1)^{5/2} - \frac{1}{6}(1)^{3/2}$$

$$\frac{1}{10} \cdot 243 + \frac{1}{6} \cdot 27 - \frac{1}{10} - \frac{1}{6} = \boxed{28.5\bar{3}}$$

$$u = 2x - 1$$

$$\frac{du}{dx} = 2 \rightarrow du = 2 dx$$

$$\frac{u+1}{2} = x$$

$$x=1 \rightarrow u=1$$

$$x=5 \rightarrow u=9$$

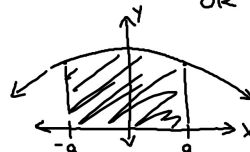
Integration of Even and Odd Functions

Let f be integrable on the closed interval $[-a, a]$.

1. If f is an even function, then:

$$\text{Recall: } f(x) = f(-x)$$

$$\text{OR } (x, y) \rightarrow (-x, y)$$



*symmetric to the y-axis.

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

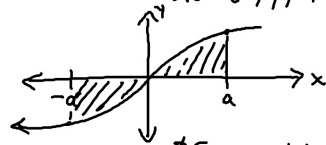
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2. If f is an odd function, then:

Recall: $f(-x) = -f(x)$

OR $(x, y) \rightarrow (-x, -y)$



* Symmetric with respect to origin

$$\int_{-a}^a f(x) dx = 0$$

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4. $\int_{-4}^4 (x^3 + 6x^2 - 2x - 3) dx$

$\int_{-4}^4 \overset{\text{odd function}}{(x^3 - 2x)} dx + \int_{-4}^4 \overset{\text{even function}}{(6x^2 - 3)} dx$

$0 + 2 \int_0^4 (6x^2 - 3) dx$

$2 [2x^3 - 3x]_0^4$

$2 [2 \cdot 4^3 - 3 \cdot 4 - 0]$

$2 (128 - 12)$

$2 (116)$

$\boxed{232}$

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HOMEWORK

pg 341 - 342; 95 - 111 odd, 119,
137, 138, 140

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